Mon. Not. R. Astron. Soc. **000**, 000–000 (1999)

# Statistical lensing of faint QSOs by galaxy clusters.

S.M. Croom and T. Shanks
*Physics Department, University of Durham, South Road, Durham, DH1 3LE, England.*

20 May 1999

**ABSTRACT**

We investigate the anti-correlation between faint high redshift QSOs and low-redshift galaxy groups found by Boyle, Fong & Shanks (1988), on the assumption that it is caused by gravitational lensing of a flat QSO number count, rather than by dust in the galaxy groups, or any other systematic effect. Using an isothermal sphere lens model, the required velocity dispersion is $\sigma = 1286^{+72}_{-91}$ km s$^{-1}$. With an isothermal sphere plus uniform density plane, the velocity dispersion is $\sigma = 1143^{+109}_{-153}$ km s$^{-1}$, while the plane density is $\Sigma_c = 0.081 \pm 0.032h$ g cm$^{-2}$. Both these values for the velocity dispersion are considerably larger than the $\sim 400 - 600$ km s$^{-1}$ expected for poor clusters and groups and imply that the mass associated with such groups is $\sim$4$\times$ larger than inferred from virial analyses. If due to lensing, this measurement clearly tends to favour high values of $\Omega_0$. We demonstrate how an estimate of $\Omega_0$ may be obtained, finding the relation $\Omega_0 = 1.3(n/3 \times 10^{-4} h^3 \mathrm{Mpc}^{-3})(r/1\ h^{-1}\ \mathrm{Mpc})(\sigma/1286\ \mathrm{km\ s}^{-1})^2$ where $r$ is the extent of the anti-correlation and $n$ is the space density of groups. In the current data systematic errors in the determination of $n$ and $r$ may dominate this measurement, but this will be a potential route to estimating $\Omega_0$ in improved galaxy-QSO datasets where these systematics can be better controlled.
We have compared our result with that of Williams & Irwin (1998) who find a positive correlation between bright LBQS QSOs and APM galaxies. Because the QSO number counts are steeper at bright magnitudes, there is no contradiction between this result and our own. Indeed, adapting the lensing analysis of Williams and Irwin to our use of groups rather than galaxies, we find that there is good agreement between the amplitude of the positive cross-correlation found for the bright QSOs and the amplitude of the negative cross-correlation found for the faint QSOs. This analysis leads to a common estimate of $\Omega_0\sigma_8 \sim 3 - 4$. This, however, is significantly higher than indicated from several other analyses. Further tests of the accuracy of the galaxy-QSO cross-correlation results and thus their implications for $\Omega_0$ and $\sigma_8$ will soon be available from the new 2dF QSO catalogue.

**Key words:** cosmology: gravitational lensing – galaxies: clustering – quasars: general

## 1 INTRODUCTION

Gravitational lensing by galaxies and clusters produces two different effects in QSO surveys. At bright magnitudes, where QSO counts are steep, a positive correlation of QSOs and foreground galaxies or clusters can be produced, as objects intrinsically fainter than the magnitude limit are amplified and hence artificially added to the sample (Gott & Gunn 1974). At fainter magnitudes, where the QSO number count slope is much flatter, it is the reduction of observed area behind the foreground lenses which dominates, producing a deficit in the background QSO number count (Wu 1994).

Here we interpret the faint QSO-galaxy group anti-correlation result of Boyle, Fong & Shanks (1988) (hereafter BFS88) (see also Shanks et al. 1983 and Boyle 1986) in terms of gravitational lensing. This result was first interpreted in terms of dust in foreground galaxies and clusters obscuring background QSOs. However, observations of galaxy groups and clusters do not show significant amounts of dust (Ferguson 1993) and the limits are at a level which make the dust hypothesis uncomfortable. Previously Rodrigues-Williams and Hogan (1994) have suggested that the anti-correlation result may be due to lensing and this is the avenue we shall pursue here. The results in this paper are based in part on those of Croom (1997).

If correct, the lensing hypothesis would allow important constraints to be placed on cosmology and large-scale structure. The deficit of QSOs near a group or cluster can be used to weigh that structure. This method has the advantage over other mass estimates in that it allows a measurement of the absolute mass of the cluster, while other estimators such as the measurement of velocity dispersions and the observation of shear due to strong lensing are effectively measuring the gradient of the cluster potential (Broadhurst et al. 1995). Other authors (e.g. Taylor et al. 1998) have looked for a deficit of galaxies behind foreground clusters to measure the lensing magnification. The advantage of using QSOs over galaxies is that they are easier to distinguish as background objects and their redshift distribution is well known. Of course, there is the disadvantage that QSOs are rare objects and so cannot be used to examine the mass distribution of individual clusters, however they can be used to investigate the properties of a distribution of clusters.

Previous searches for QSO-galaxy correlations at brighter magnitudes have produced varying results with most showing the observational evidence for QSO-galaxy associations, (see Table 1 of Wu 1994) but the statistical basis for most of the results was limited. Recently, Williams & Irwin (1998) have found a strong positive correlation between $\sim 60$ $B < 18$ LBQS (Hewett et al. 1995) $z > 1$ QSOs and APM galaxies. Below we shall compare their results with ours to check whether these two observations provide a consistent picture for the mass distribution in the Universe.

Section 2 reviews the lensing model we use in this paper. In Section 3 we compare these models to the BFS88 data. In Section 4 we compare the results of Williams & Irwin with those of BFS88. We present our conclusions in Section 5.

## 2 STATISTICAL GRAVITATIONAL LENSING

We use two analytic mass profiles to fit the observed anti-correlation; the first, and simplest, of these being the single isothermal sphere (SIS), which gives a gravitational lensing amplification of

$$A = \frac{\theta}{\theta - \theta_{\rm E}}, \quad \theta > \theta_{\rm E}, \tag{1}$$

(e.g. Wu 1994) where $\theta_{\rm E}$ is the Einstein radius, the radius within which multiple images can occur. For the SIS case this is

$$\theta_{\rm E} = 4\pi \frac{D_{\rm ls}}{D_{\rm s}} \left(\frac{\sigma}{c}\right)^2, \tag{2}$$

where $D_{\rm s}$, $D_{\rm l}$ and $D_{\rm ls}$ are the angular diameter distances from the observer to the source, the observer to the lens and the lens to the source, respectively. In our second mass profile we add a uniform density plane to the isothermal profile (SIS+plane). This could be a good approximation to the effects of clustering and large scale structure (as pointed out by Wu et al., 1996), because a distribution of isothermal spheres with an auto-correlation function of the form $\xi(r) \sim r^{-2}$ produces a uniform mass surface density. The globally measured auto-correlation function slope is $\sim -1.8$ (Davis et al. 1988), which produces a sheet of matter which is uniform to better than 10% on the scales of interest. The amplification then becomes

$$A = \frac{\theta}{\theta - \theta_{\rm E}/(1 - \Sigma_{\rm c}/\Sigma_{\rm crit})} \frac{1}{(1 - \Sigma_{\rm c}/\Sigma_{\rm crit})^2}, \tag{3}$$

(e.g. Wu et al. 1996) where $\Sigma_{\rm c}$ is the mass surface density in the plane and the critical surface density, $\Sigma_{\rm crit}$, is

$$\Sigma_{\rm crit} = \frac{D_{\rm s} c^2}{D_{\rm ls} D_{\rm l} 4\pi G}. \tag{4}$$

Gravitational lensing can cause an over- or under-density of source objects near to the lens. The ratio of observed surface density to the true surface density (unlensed) is the enhancement factor, $q$, given by

$$q = \frac{N(< m + 2.5\log(A))}{N(< m)} \frac{1}{A} \tag{5}$$

(Narayan 1989), where $A$ is the amplification factor. $N(< m)$ is the integrated number count of source objects brighter than magnitude $m$. We note that $q$ depends on the source counts fainter than the limit of the survey. With a number count of the form $N(< m) \propto 10^{\alpha m}$, we then find an angular cross-correlation function $\omega_{\rm CQ}(\theta)$ that is described by

$$\omega_{\rm CQ}(\theta) = q - 1 = A^{2.5\alpha - 1} - 1. \tag{6}$$

## 3 THE CORRELATION OF DURHAM/AAT QSOS AND GALAXY GROUPS

We look at the result from the Durham/AAT UVX Survey (Boyle 1986; Boyle et al. 1990) which shows an anti-correlation between UVX QSO candidates and galaxy groups (BFS88). This cross-correlation was carried out within 7 UKST fields, using COSMOS scans of photographic plates. Spectroscopy of the UVX catalogue (Boyle et al. 1987) suggested that with a colour limit of $u - b < -0.4$ there was $\sim 55$ per cent contamination by Galactic stars. In the BFS88 analysis the UVX criterion was tightened to $u - b < -0.5$, reducing contamination to 25 per cent while keeping 85 per cent of the QSOs. The UVX catalogue was then split into two magnitude limited samples, $17.9 < b < 19.9$ and $17.9 < b < 20.65$. The galaxy catalogue consists of all galaxies to a limit of $b_{\rm J} = 20.0$ and the cluster sample was created using a 'friends-of-friends' algorithm (Gott & Turner 1977; Stevenson et al. 1988). Groups of seven or more galaxies with density greater than 8 times the average for the field were classed as clusters, which amounted to 10 per cent of the total number of galaxies. BFS88 performed a cross-correlation analysis between the entire galaxy catalogue and the UVX sample but no significant correlation was found on any scale. Cross-correlation of cluster galaxies with the UVX catalogue resulted in negative correlations on scales $< 10'$ for both samples, the brighter sample showing a marginally more negative clustering signal. This can be interpreted as a decrease in contamination from smaller photometric errors in the brighter sample, but a second effect is that the QSO $N(m)$ slope will be steeper at this brighter limit, thus a smaller anti-correlation might be expected. Given that our results are sensitive to the exact shape and position of the break, we restrict our analysis to the fainter sample, with the proviso that in new larger samples the anti-correlation as a function of magnitude will provide an important test of the gravitational lensing hypothesis.

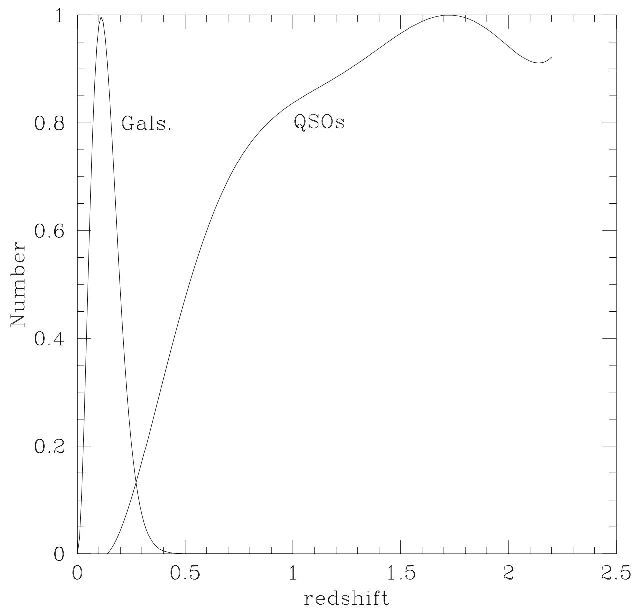


**Figure 1.** Redshift distributions assumed for the galaxy and QSO populations in the lensing models described in the text. The normalizations of the two curves are arbitrary.

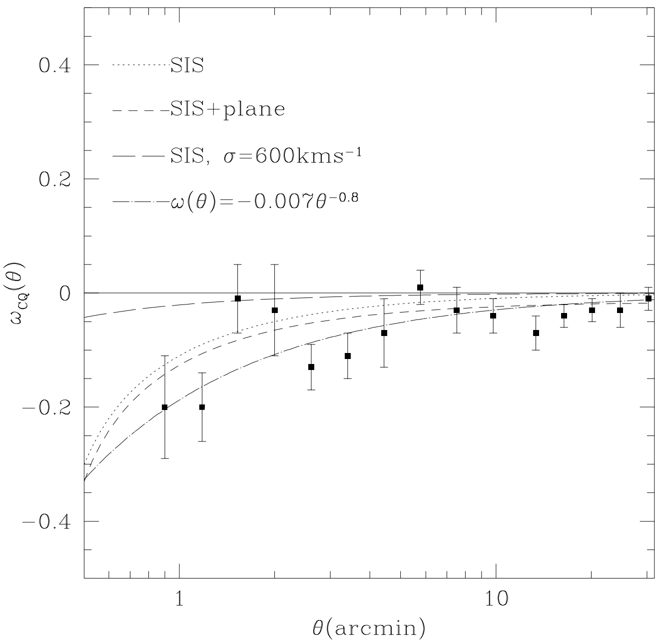


**Figure 2.** UVX-cluster cross-correlation for the faint UVX sample of BFS88 showing the best fit models; the dotted line shows the best fit SIS model, while the dashed line shows the SIS+plane model. The long-dashed line shows a lensing SIS model with $\sigma = 600$ km s$^{-1}$. Also shown as the dot-dash line is the best fit $\theta^{-0.8}$ power law.

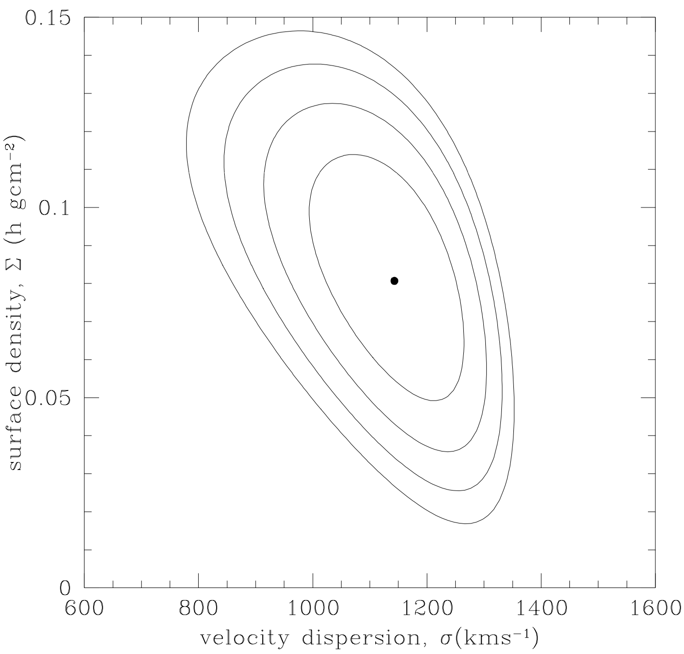


**Figure 3.** Confidence contours for the fit of the two component SIS+plane model to the cross-correlation result for the faint UVX sample, corrected for 25% stellar contamination in the QSO survey. Contours are at $\Delta\chi^2 = 1, 2, 3, 4$ ($\Delta\chi^2 = 1$ and 4 correspond to 1 and 2 $\sigma$ errors respectively).

### 3.1 A comparison of lensing models and the data

The effect of gravitational lensing is strongly dependent on the slope of the QSO number-magnitude relation at faint magnitudes. We use the number counts from Boyle, Shanks & Peterson (1988) which give an asymptotic faint end slope of $\sim 0.28$. We have confirmed that this is a reasonable representation of the integral QSO number count at $\sim 1$ mag fainter than our magnitude limit, the region from where we expect amplified QSOs to come, by using the deeper data of Boyle, Jones & Shanks (1991). A flatter slope would clearly reduce the lensing mass required. The separation of observer, lens and source also affects the lensing amplification. To take this into account in our model, we integrate the known QSO redshift distribution over the effective range of the Durham/AAT survey ($0.3 < z < 2.2$). This gives us an effective lensing amplification for a particular lens mass. For the galaxies we assume the analytic form of $N(z)$ given by Baugh & Efstathiou (1993):

$$\frac{\mathrm{d}N}{\mathrm{d}z} \propto z^2 \exp\left[-\left(\frac{z}{z_{\mathrm{c}}(m)}\right)^{3/2}\right], \tag{7}$$

where $z_{\mathrm{c}} = (0.016(b_{\mathrm{J}}-17.0)^{1.5}+0.046)/1.412$. This is shown integrated to $b_{\mathrm{J}} = 20$ in Fig. 1 along with a polynomial fit to the QSO $N(z)$ (Shanks & Boyle 1994). The two populations occupy almost completely independent volumes, less than 1% of the QSOs are at $z < 0.3$ while less than 0.5% of the galaxies are at $z > 0.3$. We assume an $\Omega_0 = 1$ cosmology throughout this analysis, but it should be noted that when the lensing mass is at low redshift ($z \sim 0.1$) cosmology has a relatively small effect as $D_{\mathrm{s}} \sim D_{\mathrm{ls}}$.

We compare the SIS lens model (from Eqs. 1 and 6) to the cross-correlation result $\omega_{CQ}(\theta)$ of the faint sample shown in Fig. 2. We have allowed for 25% contamination of the QSOs by randomly distributed stars. Using a minimum $\chi^2$ fit the velocity dispersion is $\sigma = 1286^{+72}_{-91}$ km s$^{-1}$ (reduced $\chi^2 = 1.44$). The dotted line in Fig. 2 shows this model. The SIS+plane model (Eqs. 3 and 6) is shown as the dashed line in Fig. 2, the best fit in this case has velocity dispersion $\sigma = 1143^{+109}_{-153}$ km s$^{-1}$ and the surface density in the plane of $\Sigma_c = 0.081 \pm 0.032h$ g cm$^{-2}$. The reduced $\chi^2$ for this fit is 1.07. The confidence levels for this fit are shown in Fig. 3.

The values found are significantly larger than directly measured velocity dispersions of poor clusters and groups. However, because BFS88 correlate cluster members with UVX objects, they effectively weight each cluster by the number of member galaxies. Stevenson et al., (1988) find the fraction of clusters as a function of the number of members follows an approximate power-law with a slope of −2.2. From this we can calculate the mean cluster membership, $\bar{n}$. Integrating the relationship between $n = 7$ and $n = 50$ gives $\bar{n} = 15$. However, a member weighted mean gives $\bar{n} = 20$. Thus, the BFS88 result is probing clusters which typically have $\sim 20$ members. The density on the sky of these clusters is $\sim 0.8$ deg$^{-2}$, which can be compared to the density of clusters in the APM Cluster Catalogue (Dalton et al. 1997) of $\sim 0.2$ deg$^{-2}$ and the density of richness class 0 or greater Abell clusters which is $\sim 0.1$ deg$^{-2}$ (Abell et al. 1989). Thus an 'average' cluster used by BFS88 is significantly poorer than Abell richness clusters. The velocity dispersion that might be expected for clusters of this richness is $\sigma \sim 400 - 600$ km s$^{-1}$ (e.g. Ratcliffe et al. 1998). For comparison, a lensing model corresponding to a velocity dispersion of 600 km s$^{-1}$ is shown in Fig. 2; the model is formally rejected at $> 5\sigma$. It therefore appears that the masses implied for the galaxy groups from lensing are $\sim 4$ times bigger than expected from virial analyses.

Although the addition of the uniform plane to mimic the effects of clustering of clusters helps to improve the fit to $\omega_{QC}$, Fig. 3 shows that this only reduces the velocity dispersion of the clusters for high values of $\Sigma_c$. Wu et al., (1996) find that the maximum mass likely to be associated with lenses from large-scale structure is $\sim 0.01 - 0.02h$ g cm$^{-2}$, which assumes that the matter density in clusters is near the critical density (i.e. $\Omega_{clus} \sim 1$). Values of $\Sigma_c$ in this range are only compatible with group velocity dispersions greater than 1000 km s$^{-1}$ (see Fig. 3)

We now demonstrate how the lensing estimates of average group mass, via the velocity dispersion $\sigma$, could be used to obtain a new estimate of $\Omega_0$. Using the 0.8deg$^{-2}$ sky density of groups from above, we infer an approximate space density of galaxy groups in the range n = $2 - 4 \times 10^{-4}$h$^3$Mpc$^{-3}$. The lower value comes from integrating the proper volume to z=0.1 and assuming all groups are detected to this redshift, and on the basis of Fig. 1 that this contains half the group sky density. The higher value comes from using the galaxy n(z) in Fig. 1 to derive the galaxy selection function. This is multiplied by the proper volume and the product is integrated to z=0.7, to give the effective volume from which the group space density is then derived. Multiplying by the estimated mass per group obtained by integrating the isothermal sphere profile out to a radius $r$ leads to the estimate of $\Omega_0$. We therefore find

$$\Omega_0 = 1.3 \left(\frac{n}{0.0003h^3\mathrm{Mpc}^{-3}}\right)\left(\frac{r}{1h^{-1}\mathrm{Mpc}}\right)\left(\frac{\sigma}{1286 \text{ km s}^{-1}}\right)^2, \quad (8)$$

where $r$ is now the extent of the anti-correlation, or the effective extent of the isothermal sphere. We note that the dependence on $h$ cancels out and there is no dependence on any biasing parameter. The above scaling values represent our best estimate for $n$, $r$ and $\sigma$. The value for r is obtained from consideration of Fig. 2 where $\theta \sim 10'$ corresponds to 1 $h^{-1}$ Mpc. The errors on $n$ and $r$ are unfortunately likely to be dominated by systematic components, with each potentially varying by a factor of 2. This is due to the approximate methods used to determine both the surface density of clusters, the space density of clusters, and the limiting scale of the anti-correlation. We also note that the measurement of $\sigma$ is dependent on $\Omega_0$ through the angular diameter distance terms in eq. 2, although at the redshifts considered this effect is small (a $\sim$ 25% difference in mass between $\Omega_0 = 1$ and $\Omega_0 = 0$), and may currently be dominated by more serious systematic effects. More meaningful error estimates must await larger galaxy-QSO datasets, possibly with full redshift information for the galaxies as well as the QSOs, where the extent of the anti-correlation and the group density can be better defined.

A further final problem is that this analysis for $\Omega_0$ also assumes that the groups are physically real and not significantly contaminated by accidental line-of-sight over-densities on the sky, which Stevenson et al (1988) suggested was a possibility. Of course, even accidental over-densities will act as lenses and it is not yet clear how sensitive this estimate of $\Omega_0$ is to this type of contamination. Again, in a survey which also has full galaxy redshift information, such as the forthcoming 2dF QSO/galaxy redshift survey, the effects of spurious groups could be more easily checked.

## 4 COMPARISON WITH THE LBQS-GALAXY CROSS-CORRELATION RESULT

We now compare our conclusions with those of Williams & Irwin (1998) (henceforth WI98) who have found a strong positive correlation between APM galaxies and LBQS QSOs which is significant out to scales of $\sim 60'$. These authors find that their positive correlation is an order of magnitude larger than that expected from a model with $\Omega_0 = 0.3$ and galaxy bias of $b \sim 1$, based on a comparison of the galaxy-galaxy angular correlation function and the QSO-galaxy cross-correlation function. WI98 derive the relation:

$$\omega_{QG}(\theta) \simeq (2\tau/b)(2.5\alpha - 1)\omega_{GG}(\theta). \quad (9)$$

Here $\alpha$ is the slope of the QSO number counts, $b$ is the galaxy bias, assumed to be constant as a function of scale and $\tau$ is the optical depth of the lenses:

$$\tau = \rho_{crit}\Omega_0 \int_0^{z_{max}} \frac{(c\mathrm{d}t/\mathrm{d}z)(1+z)^3}{\Sigma_{crit}(z, z_s)}\mathrm{d}z. \quad (10)$$

For a given value of $\Omega_0$, a bias value can therefore be found. This analysis can easily be applied to the faint QSO anti-correlation result of BFS88. We fit a power law with a slope of –0.8 to the auto-correlation of clusters measured by Stevenson et al. (1988), finding $\omega_{CC}(\theta) = (0.140 \pm 0.053)\theta^{-0.8}$. We then also fit a –0.8 power law to the anti-correlation between QSOs and clusters, which we find to be $\omega_{\rm CQ}(\theta) = (-0.0071 \pm 0.0059)\theta^{-0.8}$. With an assumed number count slope of $0.28 \pm 0.02$ these values then imply a value of $\tau/b = 0.085 \pm 0.077$. We assume $\Omega_0 = 1$ and integrate Eq. 10 to $z_{\rm max} = 0.2$, the redshift at which the $N(z)$ relation described by Eq. 7 falls to half its peak value, this gives $\tau = 0.021$. We therefore find a bias value of the clusters used in this analysis of $b_{\rm C} = 0.25 \pm 0.23$. If we use an $\Omega_0 = 0.3$ model, as used by WI98, then $\tau$ and therefore $b_{\rm C}$ will fall by a factor of $\sim 3$. We should note here that the errors are large, and there is some uncertainty in this procedure; if we integrate $\tau$ to where the $N(z)$ drops to 3/4 ($z = 0.16$) or 1/4 ($z = 0.24$) of its peak value we find $\tau = 0.015$ and 0.029 respectively. However even if we take the largest reasonable value of $\tau$, then $b_{\rm C} = 0.34 \pm 0.031$, which is still an order of magnitude lower than expected for clusters. Thus, in rough agreement with $b_{\rm G} \sim 0.07$ from WI98, we find a bias value estimated from statistical lensing which is an order of magnitude less than predicted by other methods. We also note that the WI98 result is consistent with the QSO-galaxy cross-correlation measured by BFS88, although BFS88 do not find a significant anti-correlation.

Although our result appears to be consistent with WI98, they are both clearly significantly out of line with other current estimates of the combination of $\Omega_0$ and bias. The space density of galaxy clusters gives $\Omega_0^{0.5}\sigma_8 \simeq 0.5$ (Eke et al. 1998) and dynamical estimates such as the measurement of redshift space distortions give similar values (e.g. Ratcliffe et al. 1998). We could possibly appeal to the scale dependence of bias to bring these different results into agreement, however, this would require an order of magnitude change in bias over a scale of $\sim 10\ h^{-1}$ Mpc. Taken at face value the above lensing result appears to suggest much more mass is present in the Universe than is detected from the distribution and motion of galaxies.

## 5 DISCUSSION AND CONCLUSIONS

BFS88 originally interpreted the UVX QSO-cluster anti-correlation as being due to absorption by dust present in clusters, the required amount of absorption being $A_{\rm B} \simeq 0.2$ mag. Ferguson (1993) finds no evidence for any reddening due to dust in clusters, and the 90% upper limit on the reddening is $E(B-V) = 0.06$. This upper limit is just consistent with the required absorption assuming $A_{\rm B} = 4.10E(B-V)$, and it is therefore still possible that lensing *and* absorption could both play a part in producing the anti-correlation result. However, it is impossible for dust in groups to also provide an explanation for the strong positive QSO-galaxy correlation found by Williams & Irwin and if their result is due to lensing then an anti-correlation is expected at faint QSO magnitudes which is comparable to that discussed here. If both results prove to be real, the simplest interpretation is that both are due to gravitational lensing.

Assuming that the measured anti-correlation is due to gravitational lensing, we find that fitting an isothermal sphere model for the cluster potentials gives a larger than expected velocity dispersion. Adding a uniform density plane to the mass profile does not significantly affect this conclusion. These lensing mass estimates suggest cluster/group masses which are $\sim 4$ times larger than expected from virial theorem analyses. We discuss a potential method to determine $\Omega_0$ from this type of mass estimate combined with a cluster/group space density measurement. We demonstrate this method with the current data, although an accurate measure of $\Omega_0$ will have to wait for larger and better controlled galaxy-QSO dataset.

We find consistency between the high $\Omega_0/b \sim 3-4$ value implied by the strong positive QSO-galaxy cross-correlation seen at bright QSO magnitudes (Williams & Irwin 1998) and the negative QSO-galaxy cross-correlation seen at faint QSO magnitudes (BFS88), if lensing is assumed to cause both effects. Applying the method of Williams & Irwin to both these cross-correlation results gives $\Omega_0\sigma_8 \sim 3-4$ (where $\sigma_8 \sim 1/b$) and the inferred values of $\Omega_0^{0.5}\sigma_8$ are therefore 6–8 times higher than those inferred from arguments based on the space-density of rich clusters.

Of course, it is still possible that some combination of systematic and random errors have contrived to produce the positive QSO-galaxy correlation seen in LBQS and the anti-correlation detected by BFS88. The importance of the above results suggests that it is vital to make further observational checks as to the reality of the QSO-galaxy cross-correlation signal. Fortunately, extended analyses of the above type will soon be possible with the completion of new large redshift surveys such as the 2dF QSO Redshift Survey and the 2dF Galaxy Redshift Survey. These two samples with 25000 QSOs and 250000 galaxies covering the same areas of sky should allow a definitive measurement of the cross-correlation function between background QSOs and galaxies at both bright and faint QSO magnitudes.

**acknowledgements**

SMC acknowledges the support of a Durham University Research Studentship. This paper was prepared using the facilities of the Durham STARLINK node.

## REFERENCES

Abell, G.O., Corwin, H.G., Olowin, R.P., 1989, ApJS, 70, 1
Baugh, C.M., Efstathiou, G., 1993, MNRAS, 265, 145
Boyle, B.J., 1986, PhD Thesis, University of Durham
Boyle, B.J., Jones, L.R., Shanks, T,. 1991, MNRAS, 251, 482
Boyle, B.J., Fong, R., Shanks, T., Peterson, B.A., 1990, MNRAS, 243, 1
Boyle, B.J., Fong, R., Shanks, T,. 1988, MNRAS, 231, 897, (BFS88)
Boyle, B.J., Shanks, T., Peterson, B.A., 1988, MNRAS, 235, 935
Boyle, B.J., Fong, R., Shanks, T., Peterson, B.A., 1987, MNRAS, 237, 987
Broadhurst, T.J., Taylor, A.N., Peacock, J.A., 1995, ApJ, 438, 49
Croom, S.M., 1997, PhD Thesis, University of Durham
Dalton, G.B., Maddox, S.J., Sutherland, W.J., Efstathiou, G., 1997, MNRAS, 289, 263
Davis, M., Meiksin, A., Strauss, M.A., Da Costa, L.N., Yahil, A., 1988, ApJ, 333, L9
Eke, V.R., Cole, S., Frenk, C.S., Patrick Henry, J., 1998, MNRAS, 298, 1145
Ferguson, H.C., 1993, MNRAS, 263, 343
Gott, J.R., Gunn, J.E., 1974, ApJ, 190, L105
Gott, J.R., Turner, E.L., 1977, ApJ, 216, 357

Hewett, P.C., Foltz, C.B., Chaffee, F.H., 1995, AJ, 109, 1499
Narayan, R., 1989, ApJ, 339, L53
Ratcliffe, A., Shanks, T., Parker, Q.A., Fong, R., 1998, MNRAS, 296, 191
Rodrigues-Williams, L.L., Hogan, C.J., 1994, AJ, 107, 451
Shanks, T., Boyle, B.J., 1994, MNRAS, 271, 753
Shanks, T., Fong, R., Green, M.R., Clowes, R.G., Savage, A., 1983, MNRAS, 203, 181
Stevenson, P.R.F., Fong, R., Shanks, T., 1988, MNRAS, 234, 801
Taylor, A.N., Dye, S., Broadhurst, T.J., Benitez, N., Van Kampen, E., 1998, ApJ, 501, 539
Williams, L.L.R., Irwin, M., 1998, MNRAS, 298, 378 (WI98)
Wu, X.P., 1994, AA, 286, 748
Wu, X.P., Fang, L.Z., Zhu, Z.H., Qin, B., 1996, ApJ, 471, 575